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RELATIVISTIC KINEMATICS FOR MOTIONS FASTER THAN LIGHT

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SUMMARY

The kinematic relations of special relativity can be extended in a simple way to motions faster than light. Being thus restricted, however, the theory does not address the question of how, or whether, such velocities might be achieved. The theory is based on the use of conformal, or "radar" coordinates in two dimensions and differs from the theory of "tachyons" in that transition to imaginary values does not occur. However, the correspondence between coordinate systems in relative motion at $V > c$ is no longer single-valued but requires a folded surface analogous to the well-known Riemann surface in the theory of conformal transformations in elliptic space. Moreover, visual observation of an object traveling at superlight speed shows not one, but two objects which appear suddenly at a point, then separate and move in opposite directions. Measurement of the velocities of these apparent objects by the Doppler method would reveal only sublight velocities. The theory shows further that velocities greater than that of light would not benefit an interstellar traveler since the twin paradox becomes inverted at such speeds.

1. INTRODUCTION

CONSIDERING THE BARRIER AT THE VELOCITY OF LIGHT in a broader context, we see not one but three such barriers, each associated with the singular behavior of the wave equation when transformed to moving coordinates. Thus the so-called "sonic barrier" appears when the factor $1/(1 - v^2/c^2)^{1/2}$ of the Lorentz transformation approaches infinity at the velocity of sound c in air. A second barrier appears when a particle approaches the (reduced) velocity of light in a transparent medium. The third and supposedly final barrier appears at the velocity of light in free space. As far as is known, no particle or object has ever exceeded this final limiting speed, although objects can and do exceed the velocity of sound in air and also the reduced velocity of light, as in the phenomenon of Cerenkov radiation.

In the aerodynamic case the approximate nature of the theory is easily recognized and the sonic barrier takes on a finite height when a more exact, nonlinear theory is employed. In Cerenkov radiation, a particle moving at less than C in one medium suddenly enters a medium in which the velocity of light is reduced, thus violating the assumption of constant v/c . The error involved in the assumption of unchanging velocity is discussed at length by Heaviside [1] in his theory of electrons moving at superlight velocities. Nevertheless, in experiments in which particles are accelerated by electromagnetic means, the particles have never reached the velocity c . Indeed Rosen [2] has shown that such processes could not achieve even the reduced velocity of light in a transparent medium. In spite of this a study of the kinematics of motions faster than light may not be without interest, since it brings out the sometimes arbitrary nature of the discussions often used in conventional theory. Moreover, our analysis shows that such

velocities, even if they could be achieved, would not benefit an interstellar traveler.

2. THE USE OF CONFORMAL COORDINATES IN RELATIVISTIC KINEMATICS

Special relativity acquires its simplest form when expressed in terms of conformal coordinates in two dimensions. Such coordinates are in reality nothing more than measurements made by the conventional radar technique. Thus a spacecraft A at rest at the origin of the x, t , system determines the distance to a point x_1 by transmitting a pulse at a time t_0 and noting the time t_2 of the return. The distance x_1 is simply;

$$x_1 = 1/2(t_2 - t_0) \quad (1)$$

Likewise the time t_1 assigned to the point x_1 is

$$t_1 = 1/2(t_2 + t_0) \quad (2)$$

Here we have chosen units in which the velocity of light is one. A second spacecraft B considered at rest in the system x', t' makes measurements in exactly the same way. To establish a regular system of coordinates, A and B are supposed to emit radar signals at a constant frequency with the aid of accurate oscillators or clocks.

The equations describing the course of such radar signals in either the A or the B systems are of the form

$$x \pm t = \text{constant}$$

and are represented by lines at 45° to the axes. Thus, we assume that the velocity of light is constant, unaffected by the motion in either system. As emitted by oscillators at a constant, unit frequency such lines will appear with equal spacing in each system. However, if B is in motion

relative to A the B signals will appear at a Doppler-shifted frequency in the A system. Figure 1 shows radar signals emitted by B and the associated coordinates x' , t' as they appear when mapped on the A system. Here we have assumed that B moves away from A, then reverses its motion and returns. In spite of the motion, B remains at the center of the expanding radar pulses, as may be seen by counting the lines $x' = \text{const.}$ on either side. As thus represented in the A system, such coordinates show considerable distortion. In the B system itself they are, of course, a perfectly rectangular system. Such diagrams are helpful in understanding the analytical relations involved in the theory.

We may represent the correspondence between the x' , t' and the x , t systems very simply by the equations

$$\left. \begin{aligned} x' + t' &= F(x + t) \\ x' - t' &= G(x - t) \end{aligned} \right\} \quad (3)$$

If we write

$$f = \frac{dF}{d(x + t)} ; \quad g = \frac{dG}{d(x - t)} \quad (4)$$

we find that the functions f and g are simply the Doppler-shifted frequencies observed in the A system corresponding to the unit frequency emitted by B. Such Doppler shifts are commonly used to measure velocities, and after a little algebra we find, for the velocity of B in the A system,

$$V = \frac{g - f}{g + f} \quad (5)$$

If we now introduce a third system C, designate the velocity of B relative to A as V_1 , and the velocity of C relative to B as V_2 , we find after two successive transformations (3) the following formula for the

velocity V_3 of C relative to A:

$$V_3 = \frac{g_1 g_2 - f_1 f_2}{g_1 g_2 + f_1 f_2} = \frac{V_1 + V_2}{1 + V_1 V_2} \quad (6)$$

This relation is often called "the relativistic law for the addition of velocities." However, we have made no use of the principle of relativity, but have used merely the constancy of the velocity of light. A better designation would be "the law of composition of velocities in conformal coordinates."

As Eq. (6) shows, the velocity of light plays the role of a singular, invariant velocity to which no other velocity, either greater or smaller, can be added.* Figure 2 is a diagram plotted from Eq. (6); it shows graphically the singular nature of the velocity $c = 1$. I have extended this diagram beyond its usual range to illustrate the relation for super-light velocities (see [3]).

If we differentiate Eqs. (3) and then multiply we find that

$$dx'^2 - dt'^2 = fg(dx^2 - dt^2) \quad (7)$$

If the coordinate systems A and B are to be completely equivalent, the transformation and its inverse must have the same scale. We then have $fg = 1$ and Eqs. (3) reduce to the Lorentz transformation. However, complete equivalence cannot be maintained between systems in which one or the other undergoes acceleration. If we identify B as the accelerated system and A as an inertial system, then we may establish a local

*It may be observed that if a measurement in space could be made by sending signals at some other velocity, say c' , then the new signal velocity would become the invariant. However, experiments made by independent methods show that c , not c' , is the invariant velocity.

equivalence by making fg equal to 1 in the vicinity of the "world line," $x' = 0$ of B. The following examples will clarify this relation.

To show the simplicity of our method we consider next the well-known problem of motion with constant acceleration (see [4]). Since $1g$ is very nearly 1 light-year per year per year, we suppose that B undertakes a trip to a distant star at the convenient acceleration of $1g$. The equations connecting the x', t' and the x, t coordinates in this case are:

$$\left. \begin{aligned} e^{x'+t'} &= x + t \\ e^{x'-t'} &= x - t \end{aligned} \right\} \quad (8)$$

Multiplying the two equations, we obtain

$$e^{2x'} = x^2 - t^2 \quad (9)$$

The path of B is given by the equation of the line $x' = 0$ and is clearly

$$x^2 - t^2 = 1 \quad (10)$$

that is, a rectangular hyperbola with the velocity V asymptotically approaching the speed of light (see Fig. 3). In order to determine the elapsed time during the trip we divide the first equation by the second and obtain

$$e^{2t'} = \frac{x + t}{x - t} \quad (11)$$

Along the "world line" $x' = 0$ we have $(x + t)(x - t) = 1$, hence $1/(x - t)$ is equal to $(x + t)$ along this line. Then

$$e^{t'} = x + t \quad (12)$$

If the star is very distant, the path of B will be near its asymptote so that $(x + t) \cong 2x$; therefore,

$$t' \approx \ln 2x \quad (13)$$

For 50 light-years the time required is approximately 4.6 years, in agreement with the usual result.

If we differentiate Eqs. (8) and then multiply we find

$$e^{2x'}(dx'^2 - dt'^2) = dx^2 - dt^2 \quad (14)$$

Adopting a conventional description, we see that the B coordinate system is characterized by a gravitational field corresponding to the (unit) acceleration. Thus B, considering himself at rest in the x' , t' system, observes the remainder of the universe falling away in the negative x' direction under the apparent influence of a gravitational potential,

$$\phi = -x' \quad (15)$$

We are thus led to the well-known scalar or conformal gravitational metric (see [5])

$$e^{-2\phi}(dx^2 + dy^2 + dz^2 - dt^2) \quad (16)$$

where ϕ is the analogous Newtonian potential. Such a scalar field can be thought of as an approximation to the more general gravitational metric which may contain as many as 10 components.

Returning to our hyperbolic motion we observe that the potential ϕ vanishes in the vicinity of the world line $x' = 0$, hence the transformation reduces to locally tangent Lorentz transformations along this line, consistent with the assumption that A is an inertial system.

Figure 4 shows an example in which B moves away from A, then reverses his velocity and returns. Such examples are easily constructed by taking suitable values for the Doppler functions f and g . Here we take $g = 2$ and $f = 1/2$ for the outgoing portion, with these values

interchanged on the return. According to Eq. (5) the velocities are $\pm 3/5$. The condition $fg = 1$ results in an elapsed time for the B system of 8 years, while the time in the A system is 10 years. As the figure shows, inverting the transformation discloses a "distance paradox" in addition to the well-known time paradox. A does not move as far to the left of B as B moves to the right of A. Such differences are easily explained by noting that the radar signals emitted by B during a portion of his outward journey are not returned until he is well on his way back. The lack of equivalence between the direct and the inverse transformations is a characteristic feature of radar measurements made from systems in accelerated motion.*

3. CONFORMAL COORDINATES AT SUPERLIGHT VELOCITIES

In the theory of tachyons it is supposed that the factor $(1 - v^2)^{1/2}$ of the Lorentz transformation becomes imaginary at superlight velocities. It is not difficult to show, however, that this factor changes discontinuously to the real value $(v^2 - 1)^{1/2}$ on transition through the speed of light (see [3]).

Set $dx' = 0$ in Eq. (7) and let $dx = V dt$; then

$$dt' = [fg(1 - v^2)]^{1/2} dt \quad (17)$$

*It will also be noted that the conformal coordinates determined by B show displacements at negative times, long before the acceleration begins. Again the explanation is a simple one. Signals emitted by B when he is at rest are not received back until his motion has begun. It is interesting that such "incoming gravitational waves" appear, even in exact solutions of the equations of general relativity (see [6]).

Now consider a series of motions of B at progressively increasing velocities, as shown in Fig. 5. The Doppler-shifted frequency g on the advancing side will increase without limit as B approaches the speed of light. The frequency f on the receding side, satisfying the condition $f = 1/g$, will approach zero in a regular fashion. However, as soon as B exceeds the velocity of light the signals g reverse their order, the last signal arriving first. The function g thus switches from positive infinity to negative infinity on passing through the singular velocity. It is during this transition that the scale factor fg changes discontinuously from $+1$ to -1 . Setting $fg = -1$ in Eq. (17) we obtain;

$$dt' = (v^2 - 1)^{1/2} \quad (18)$$

for velocities greater than 1. Figure 6 shows the easily traced course of the functions f and g .

It is interesting that the conventional measurement of velocity, relying on the absolute magnitude of the Doppler shift, would not disclose the superlight motion of B but would assign the sublight velocity $1/v$. If B moves at superlight velocity his radar pulses will trail behind and fill an expanding cone, a phenomenon well known in the case of supersonic flight and also in the case of Cerenkov radiation. In the B system, however, we must assume that such pulses remain centered on the position of B , in accordance with the invariance of the velocity of light (see Fig. 7). That such pulses could be used to measure distances in the region ahead of B is of course not an intuitive concept. However, in conventional theory we are required (by the evidence) to believe that waves travel at full speed ahead of B , even though he may be going at 90% of the speed of light. The present concept is not essentially different. Clearly,

however, the trailing light cone will complicate the relation between the x', t' and the x, t systems. The coordinate for B must preserve his position at the center of his radar pulses, while in A coordinates he moves ahead of them. The correspondence between the two systems is thus not a simple variant of the Lorentz transformation, but is multivalued. The map of B coordinates must be folded to fit on the A system. Such a folded sheet may be thought of as the analogue, in hyperbolic space, of the well-known Riemann surfaces, which are much used in the theory of complex variables, that is, conformal transformations in elliptic space.

Figure 8 shows an example in which B moves away from A at a velocity of $5/3$ the velocity of light and then stops. The folds along the lines $x - t = \text{const.}$ keep the traveler at the center of his radar pulse emitted at the time $t = 0$. This may be verified by counting the number of lines x' to the right and to the left of B as they are identified by the dots on the figure. Figure 9 shows another example in which B moves to the right of A at $V = 5/3$, reverses his velocity, and returns. Comparing this figure with Fig. 4 we find that the elapsed time in the A system is much reduced by the higher velocity, as expected. However, the elapsed time for the traveler B is not reduced but remains the same as before, 8 years. If B had acquired a still greater velocity, his time for the journey would have been even greater. The path of A in the non-inertial B system shows again the lack of uniformity in the superlight transformation. Two points in the B system correspond to a single point in the A system.

Interestingly, a body moving at superlight speeds actually appears visually as two bodies. Moving at speeds less than c , the waves emitted by B will form eccentric circles with their centers trailing behind.

Hence, when viewed from A the position of B will appear behind his true position, that is, at the center of a displaced circle. This is the well-known phenomenon of aberration. Aberration takes an extreme form when B moves faster than light, as shown by Fig. 10. Here we observe that every point in the light cone trailing B is marked by the intersection of two spherical waves. An observer in A, momentarily at such an intersection, sees two objects B_1 and B_2 at the centers of the waves. At the first encounter, B appears single, then splits into two bodies which then separate moving in opposite directions.

As noted earlier, moving faster than light does not benefit a space traveler. To obtain the elapsed time for a trip at constant velocity to a point x , set $t = x/V$. Then

$$t' = \frac{(V^2 - 1)^{1/2}}{V} x \quad (19)$$

At infinite velocity, the factor $(V^2 - 1)^{1/2}/V$ reduces to 1. Hence a trip at infinite velocity requires the light time for the traveler. Although it is made instantly in the inertial system. Einstein has remarked that motion at the velocity of light is equivalent to infinite velocity for the traveler. We now see that the converse is also true: motion at infinite velocity is equivalent to the velocity of light for the traveler.

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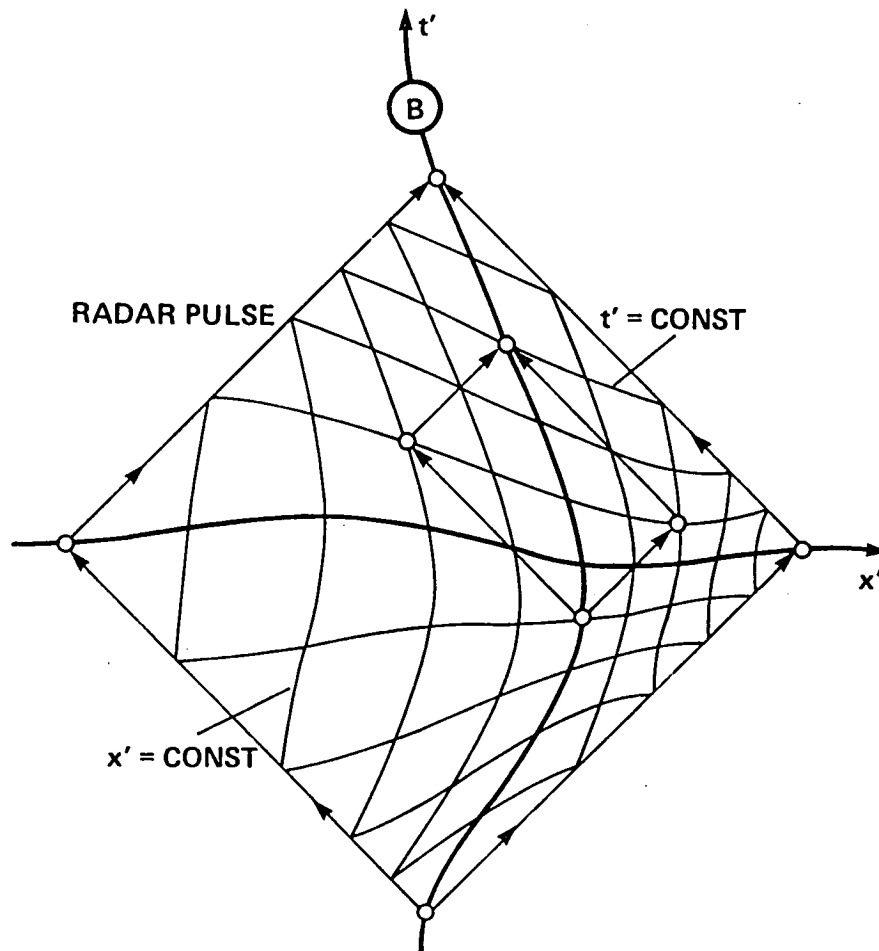


Fig. 1. Space-time coordinates determined by radar measurements from spacecraft B. Note that B remains at the center of the expanding radar pulse in spite of his motion.

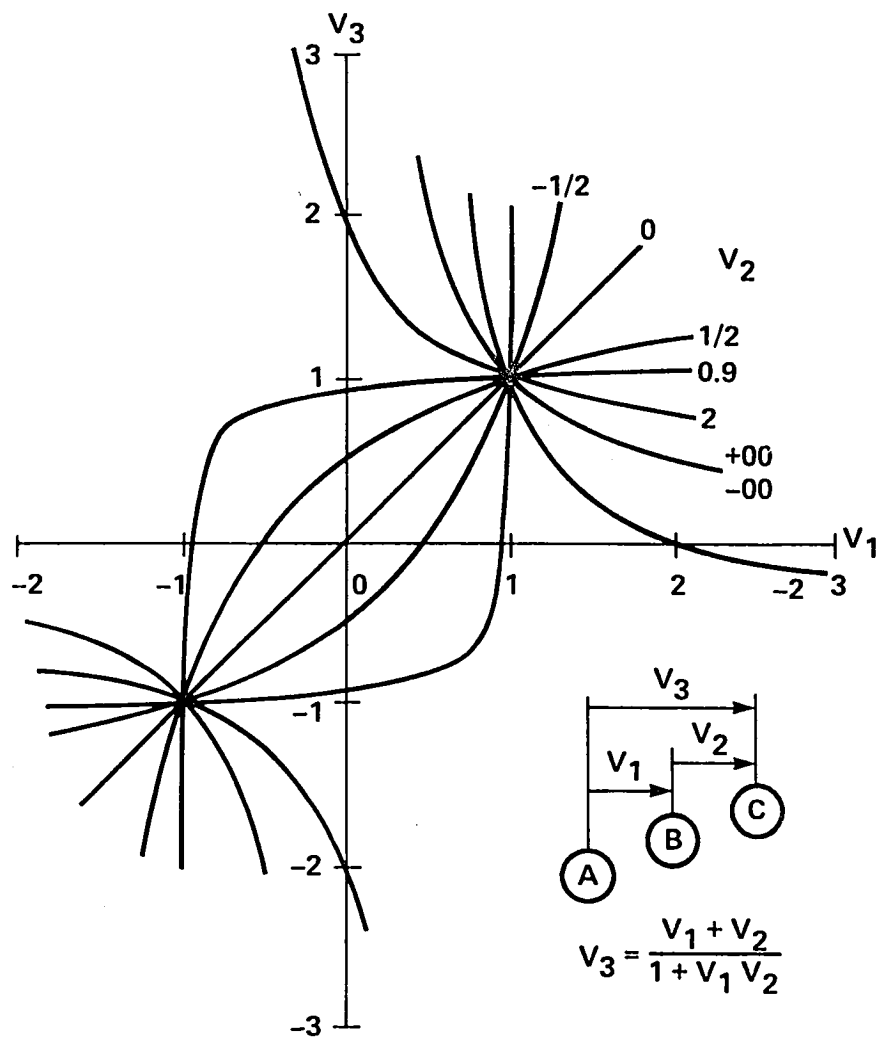


Fig. 2. Composition of velocities in conformal coordinates. No velocity, larger or smaller, can be added to the velocity of light.

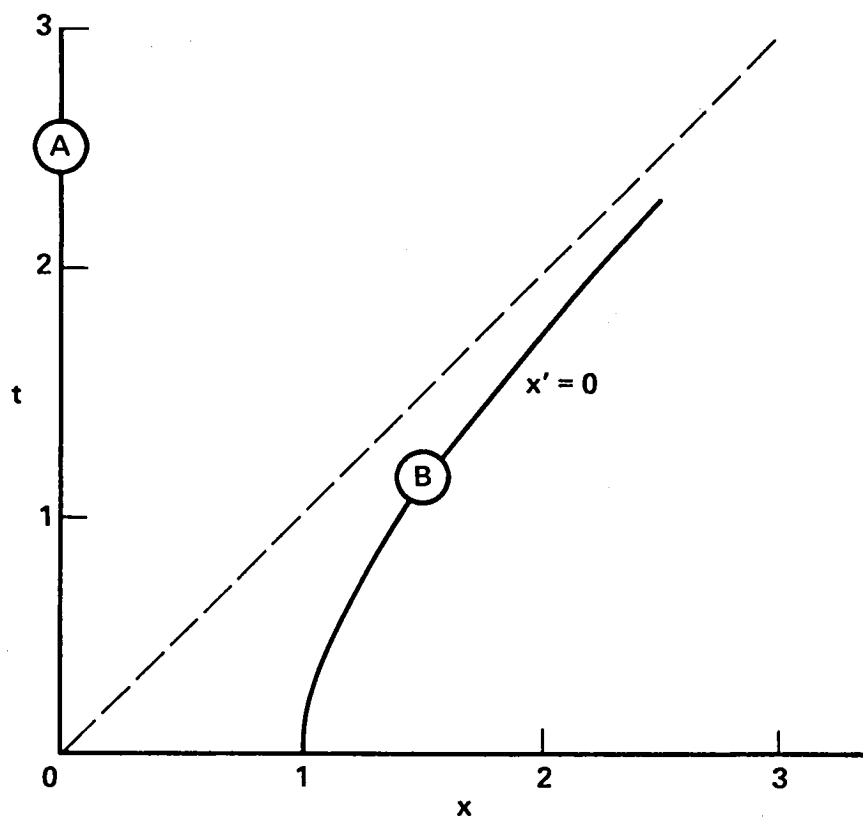


Fig. 3. Hyperbolic motion. Moving with an acceleration of 1 g, B approaches the velocity of light asymptotically.

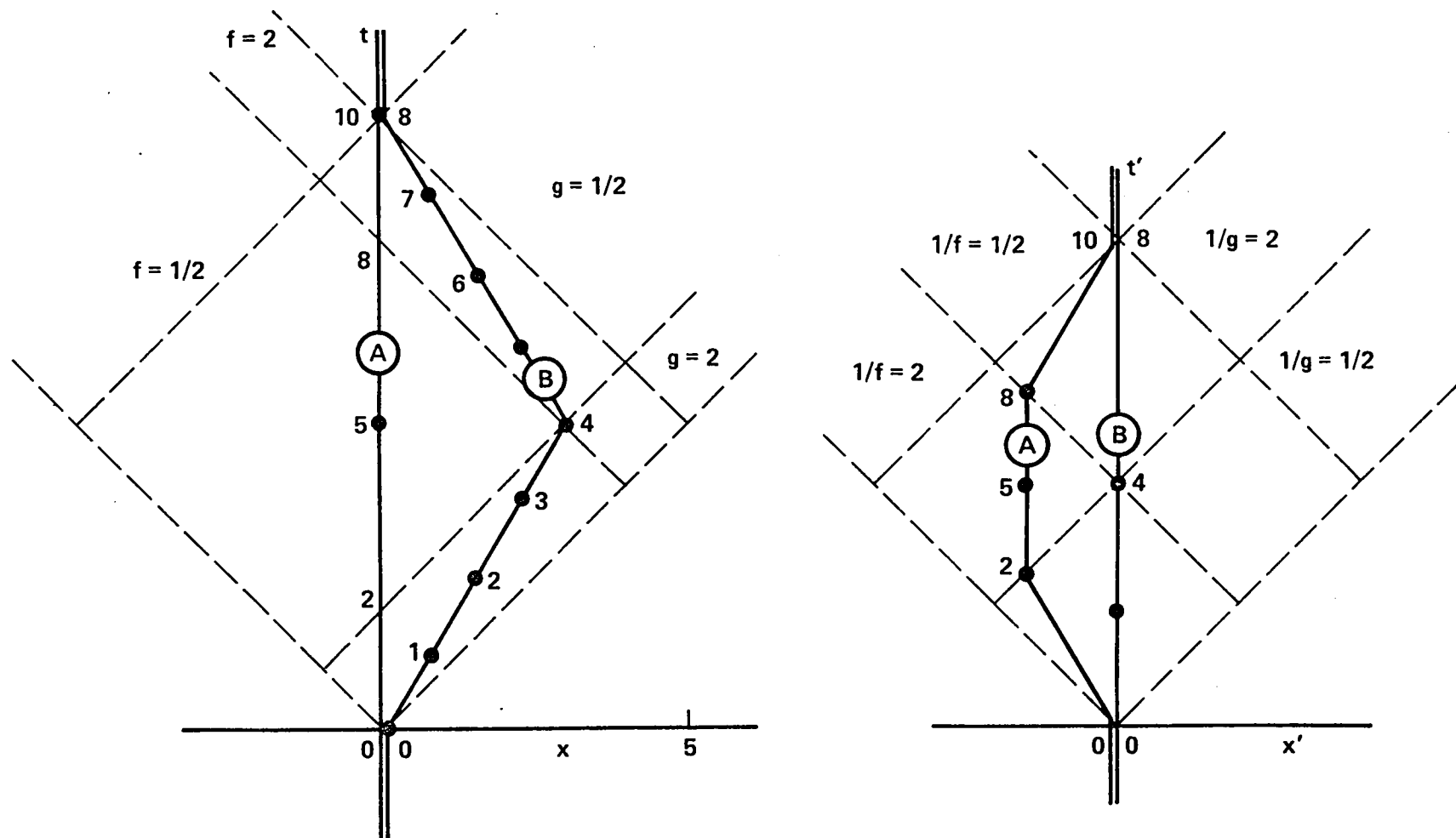


Fig. 4. Direct and inverse transformations at $V = \pm 3/5$. Because of the gravitational field in the B system the direct and inverse transformations are not similar. When B returns to A the elapsed time in his system is 8 years, while the elapsed time in the inertial A system is 10 years. Moreover, the distance from A to B is not the same as the distance from B to A.

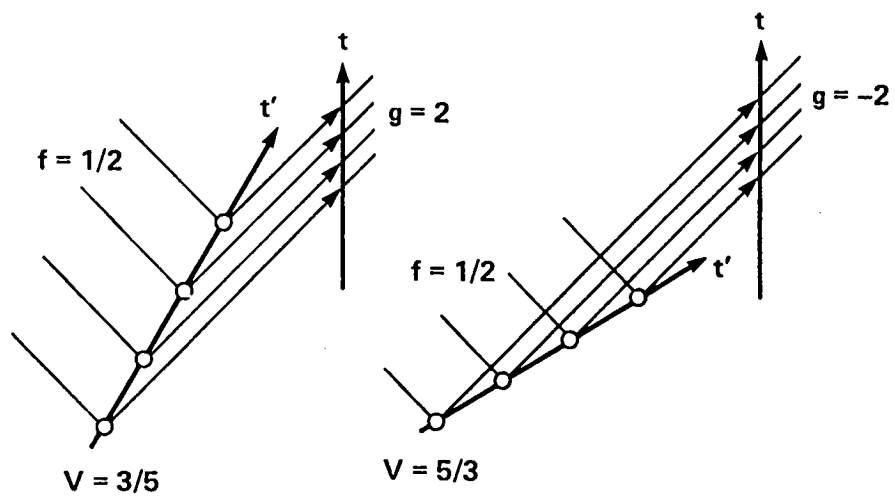


Fig. 5. Doppler shift of time signals below and above the speed of light.
 Note that a measurement of velocity by the Doppler method would not reveal the superlight velocity, but would show $V = 3/5$ in each case.

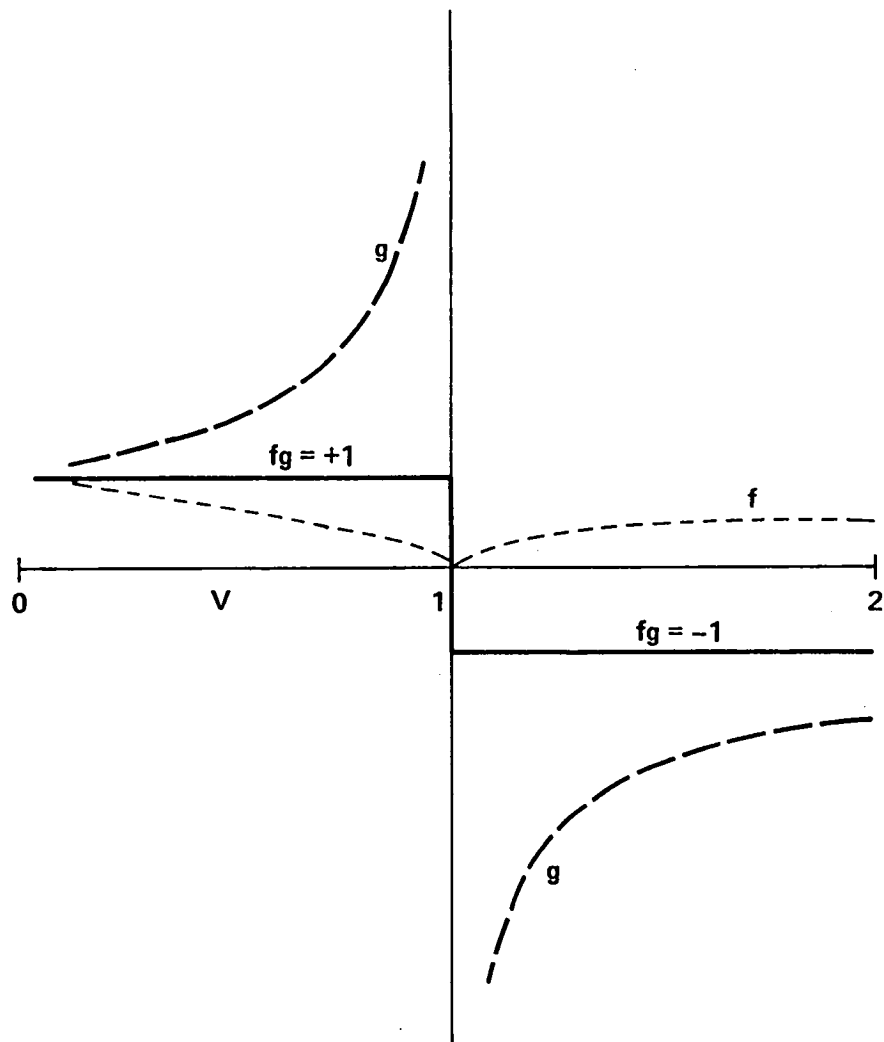


Fig. 6. Behavior of the Doppler frequencies f and g on transition through the speed of light. When the conformal scale factor fg changes to negative values the quantity $(1 - v^2)^{1/2}$ changes to $(v^2 - 1)^{1/2}$ and remains real.

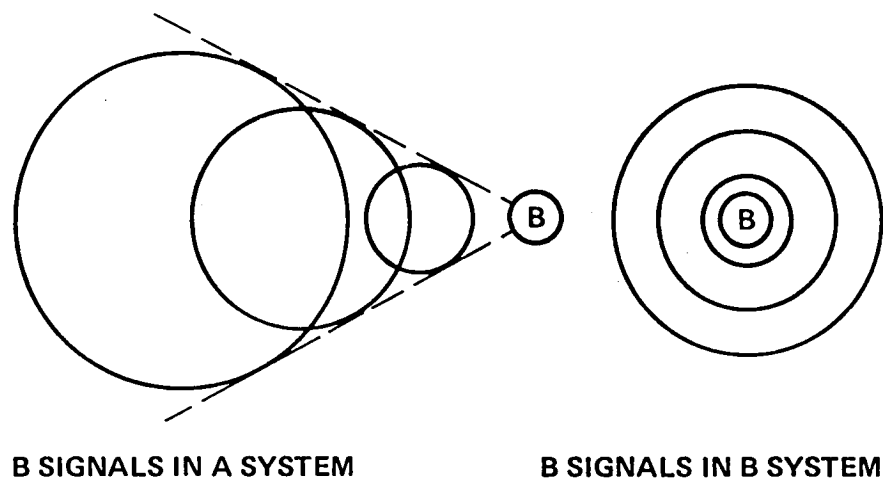


Fig. 7. Radar signals at superlight velocities.

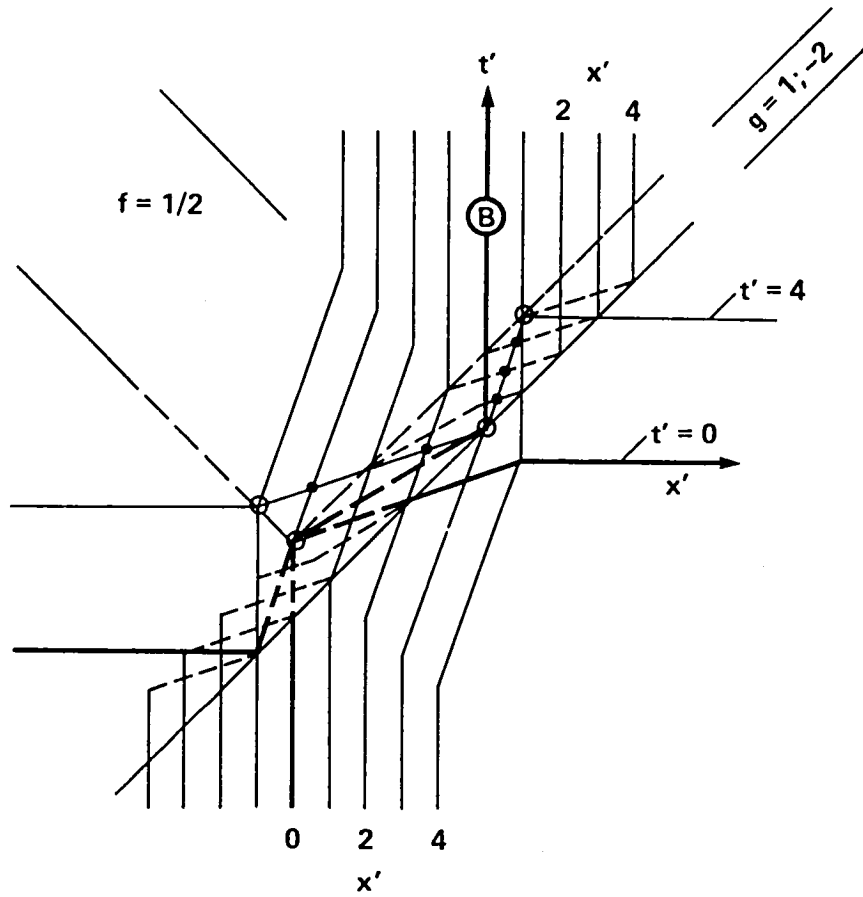


Fig. 8. Conformal transformation, $V = 5/3$. At superlight speeds the x' , t' map must be folded to fit on the x, t system.

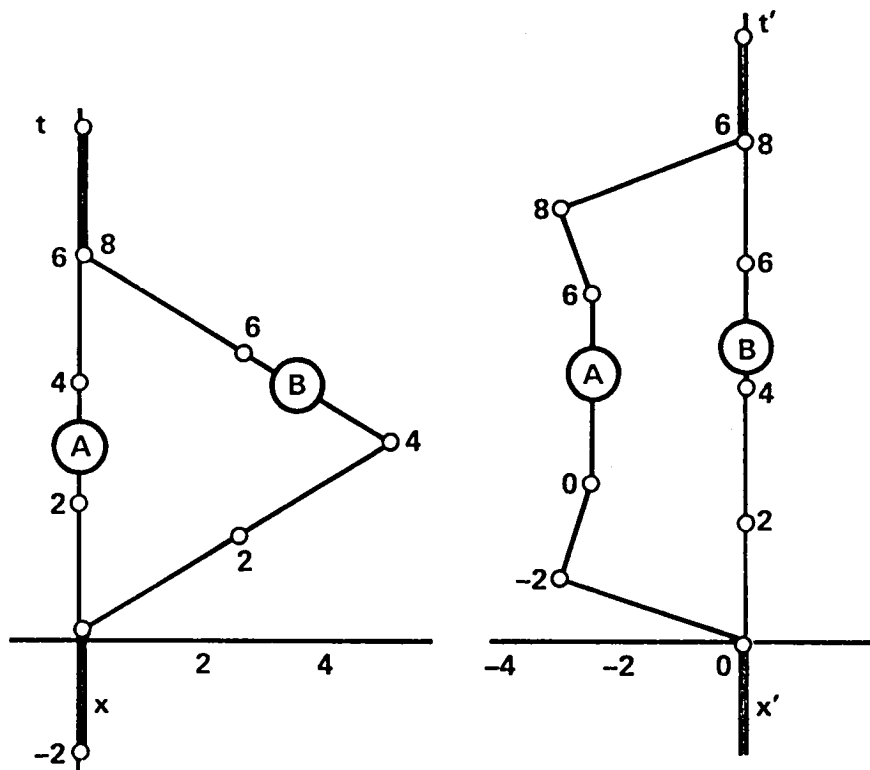


Fig. 9. Direct and inverse transformations for $V = \pm 5/3$.

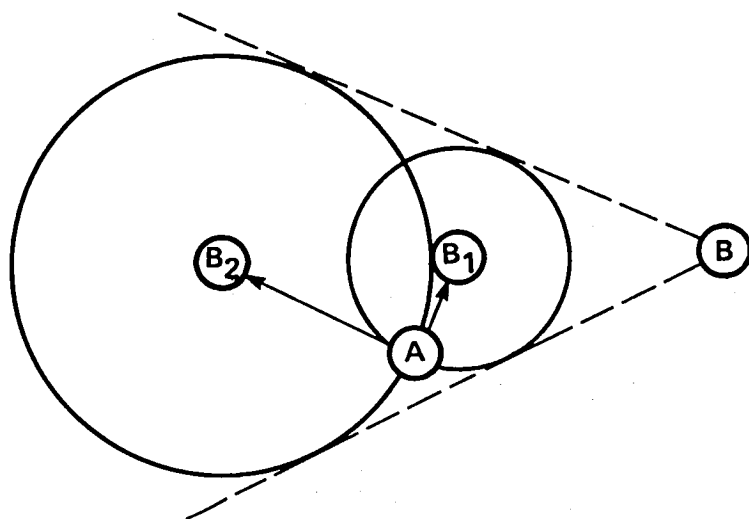


Fig. 10. Double appearance of body traveling at superlight speed. Traveler A detects two objects, B_1 and B_2 , each traveling at less than the speed of light.

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16. Abstract The paper illustrates the use of conformal coordinates in relativistic kinematics and provides a simple extension of the theory of motions faster than light. An object traveling at a speed greater than light discloses its presence by appearing suddenly at a point, splitting into two apparent objects which then recede from each other at sublight velocities. According to the present theory motion at speeds faster than light would not benefit a space traveler, since the twin paradox becomes inverted at such speeds. In Einstein's theory travel at the velocity of light in an inertial system is equivalent to infinite velocity for the traveler. In the present theory the converse is also true; travel at infinite velocity is equivalent to the velocity of light for the traveler.			
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